along with curves corresponding to equations (7c)–(7e). This figure shows that the arithmetic mean, k_a, is an appropriate upper bound for the numerical data rather than k_u, whereas the prediction of k^* underestimates the results. This result is not surprising since the effect of the boundaries has been ignored. Indeed, it is reasonable to assume that the insulating side boundaries imposed in the numerical simulations will tend to increase the effective conductivity relative to a domain that is unbounded in the transverse direction. Thus, k^* acts as an approximate lower bound of the effective conductivity while k_a remains as the upper bound. These bounds may be compared with $1-1.63V$, the best straight-line fit of the numerical data [1]. It is interesting to observe that k_u gives, at first sight, a reasonable fit of the numerical data, although one may question whether the curvature is correct.

In conclusion, the finite, square domain with circular inclusions studied by Muralidhar [1] violates the assumptions of classical effective medium theory. In particular, the latter is based on the assumptions that the averaging volume is large compared with that of the inclusion, and that the number of inclusions within the medium is large. The results in Fig. 1 show, however, that the classical theory provides useful bounds on the numerical results. In the absence of other information, it appears that k^* is a reasonable estimator of the effective conductivity. Finally, the conclusion reached by Muralidhar [1] that the static effective conductivity can be used for unsteady problems corresponds with a similar conclusion for unbounded groundwater flow domains reached by Dagan [5].

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Comments on "Analysis of close-contact melting for octadecane and ice inside isothermally heated horizontal rectangular capsule"

The purpose of this letter is to point out that the thin liquid film analysis reported in ref. [1] is a special case of a more general theory published almost three years ago [2], which was apparently overlooked. Reference [2] described melting on a rectangular contact surface, with or without relative motion (sliding) between the two solid parts, and with or without heating due to viscous dissipation in the liquid film. The analysis of Hirata *et al.* qualifies as a special case of ref. [2] for three reasons. They assumed that:

(i) The rectangular contact surface is infinitely wide (the short side was labeled W in their Fig. 4).

(ii) There is no relative motion, that is, the phase-change material does not slide laterally.

(iii) Viscous dissipation in the lubricating film is negligible.

Hirata *et al.*'s key theoretical result—the film thickness formula (17)—is essentially the same as equation (21) in ref. [2]

$$\frac{h}{L} = \left[\frac{Ste}{P_n/(\alpha\mu/L^2)}\phi\right]^{1/4} \tag{21}$$

in which h is the film thickness, L the short side of the contact surface (Hirata *et al.*'s W), Ste the liquid Stefan number, α the thermal diffusivity, μ the viscosity, and $\phi = 1$ the factor accounting for the infinitely wide shape of the contact area. When the contact area is a rectangle with the same L but finite width, the factor ϕ is smaller than 1 (Fig. 2 in ref. [2]).

It is important to note that equation (21) is expressed in terms of the instantaneous average pressure (P_n) maintained between the phase-change material and the flat heater. In this way the results of ref. [2] are applicable to any geometry in which the instantaneous average pressure may change with time, for example, because of the finiteness of the solid block of phase-change material, and the size and shape of the capsule (as in Hirata *et al.*'s geometry).

Equation (21) stresses the fact that the contact melting process is *quasisteady*, i.e. decoupled from the other time-dependent features of the greater system. In this sense, the presence of time as a variable on the right-hand side of Hirata *et al.*'s film thickness formula (17) is misleading: the time-dependence entered that expression only through the instantaneous average pressure, which changes slowly with time.

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